Workbook

Course: *Quantitative Finance   2017/2018*

1 – Simple Interest rate

1.1 Zeta Enterprises deposited €7,000.00 into a 4-year certificate of deposit paying 8% interest. Using simple interest find:
   a) The future value.
   b) The total amount of interest.
   c) The amount of interest in the last year of the deposit.

1.2 Mr. Thomas invested €18,000.00 for five and a half years. Knowing that the semiannual interest rate was 2%, find the maturity value and the total amount of interest at the end of the term.

1.3 Mr. Tobias invested 65,350.00 Euros in a financial product with a semiannual interest rate of 4.6% (simple interest). How long should he keep this investment if he wants an interest of 25000.00 Euros? (Bankers' rule)

1.4 Using simple interest, find the maturity value of 11,000.00 Euros under the following conditions:
   a) 3 years with an interest rate of 3%
   b) 8 and a half years with a semiannual interest rate of 1.9%
   c) 110 days with a quarterly interest rate of 4% (exact method)
   d) 190 days with a 4-month interest rate of 6% (exact method)
   e) 10 months with a monthly interest rate of 0.9%

1.5 On 1/1/2003 you deposit €7,000.00 at 8% into an account that pays interest on 1/1/2004 (simple interest). On 1/8/2003 you need 5,000.00 to pay off a school bill and you find a broker on e-commerce.com that will lend you the money at 12%. Should you get the loan or should you withdraw the money from your money account? How much do you save using the best method?

> Rules for your decision:
- If you withdraw any money in your account early, you will lose all of the interest on the part withdrawn.
- You can repay the loan after the money account pays its interest since you can withdraw the money without penalty.
- Compute all interest with maturity date 1/1/2004.

1.6) A debt of 42,000.00 Euros is going to be paid (principal + interest) in 4.5 years. The interest rate is:
   ➢ 7.5% in the first year
   ➢ 5% in the second year
   ➢ 4% in the third year and following periods.
   Find the total amount to be paid, using simple interest.

1.17) On 2/11/2007 Tom invests €1,000.00 at 5% per annum (simple interest). Find the interest and the maturity value of the investment on 2/4/2009.

1.19) On February 3, Vicky lends her brother James €10,000.00. James pays off the loan on October 3 with 11,000.00 Euros. Find the annual interest rate (simple interest).

1.21) On 1/1/2003 you deposit 7,000.00 Euros at 8% into an account that pays interest on 1/1/2004 (simple interest). On 1/8/2003 you need 5,000.00 to pay off a school bill and you find a broker on e-commerce.com that will lend you the money at 12%. Should you get the loan or should you withdraw the money from your money account? How much do you save using the best method?

> Rules for your decision:
> ➢ If you withdraw any money in your account early, you will lose all of the interest on the part withdrawn.
> ➢ You can repay the loan after the money account pays its interest since you can withdraw the money without penalty.
> ➢ Compute all interest with maturity date 1/1/2004.

1.9) Using simple interest, find the equivalent rates:
   a) $i_A=12\%$ => $i_S=?$
   b) $i_Q=1,2\%$ => $i_A=?$
   c) $i_M=0,25\%$ => $i_{8M}=?$
   d) $i_{2A}=12,6\%$ => $i_{4M}=?$
   e) $i_Q=4\%$ => $i_{4M}=?$
   f) $i_B=1,8\%$ => $i_{7M}=?$

1.10 (2A)) Today is New Year's Day. In return for payments of 1500 at the end of January, February, and March, and of 3000 at the end of May, July, and September, an investor agrees to pay now the total value of the 6 payments, and to either make or receive an additional payment at the end of December. Find the amount of that additional payment if it is known that the nominal annual interest rate is 6%, composed monthly. (First set up an equation of value.)

   (a) Find the smallest nominal rate of interest convertible monthly at which the accumulated value of 15,000 at the end of 3 years is at least 24,000.

   (b) Find the smallest nominal rate of discount convertible semi-annually at which the accumulated value of 15,000 at the end of 3 years is at least 24,000.

2.1 Decatur Country Chamber of Commerce issued discount notes to finance a downtown renovation project. Each discount has a face value of $2000 and a term of 10

years. The notes were sold at a simple discount rate of 4.5%. Find de proceeds of each note sold. Assume exact interest rate.

2.2) Find the maturity value of a 9-month discount note if the discount is $70 and the discount rate is 11.9%. Assume exact interest rate.

2.3) Lynn´s Inc. signed $10,000 discount note with a term of 120 days and received proceeds of $9,500. What is the simple discount rate for this note? Assume exact interest rate.

2.4) Elaine loans her brother €150 and he pays her back €160 ninety days later. Find the simple discount rate.

2.5) A €2,000 maturity value with a 180-day term is sold at a simple discount interest rate of 7.99%. Find the simple interest rate that would be equivalent to the state simple discount rate. Assume exact interest rate.

2.6) Christy will receive a €1,500 pay check in 14 days. However, she needs money now. A payday lender offers to give her today for a fee of 2% of the check amount. Find the equivalent simple discount rate and the equivalent simple interest rate.

2.7) An annual subscription to your favorite magazine costs €45.99. Although subscription renewal isn't due for another three months, the publisher is offering you a 5% off the regular price if you renew now. What would be the simple interest rate?

2.8) On May 3, 2004 Valera bids 96.2 on 182-day $500,000 T-bill.
  a) Find the discount and as an interest rate, present value and future value.
  b) On august 5, 2004 Valera sells the T-Bill to an investor desiring a 6% simple interest rate return on her investment. Find the amount the investor paid for the T-bill on a simple interest basis.

2.9) A €5,000 face value note has a term of 219 days (exact interest). The simple discount rate is 9.375%. Find the proceeds of the note.

2.10) A 3-month note is discounted by €28.75. The simple discount rate is 5.75%. Determine the maturity value and the proceeds of the note.

2.11) A €10,000 T bill with 182 days to maturity sold at auction for €9,753.16. What is the simple discount rate?

2.12) The City Council Demos €4.959,247 on March 7, 2005, in anticipation of receiving a state aid payment of €5,000,000. The loan was based on a simple discount rate of 4.25%. On what date will the City Council receive its state aid?

2.13) An investment manager is weighing a choice between two possible investments for a fund that he manages. He originally had planned to invest in a €10.000 face value, 9-month simple discount note issued by the XPTO Company, which he was offered at a simple discount rate of 8.0%. On the other hand, the company has offered to borrow the same amount of money from his fund by signing a note carrying a simple interest rate of 8.25%. Which is the better deal for the investment fund?

Part II - Compound interest - Exercises with effective rates

1.1) Zeta deposited €7,000.00 into a 4-year certificate of deposit paying 8% interest. Using compound interest find:
a) The future value.
b) The total amount of interest.
c) The amount of interest in the last year of the deposit.

1.2) Mr. Thomas invested 18,000.00 Euros for five and a half years. Knowing that the semiannual interest rate was 2%, find the future value and the total amount of interest at the end of the term.

1.6) A debt of 42,000.00 Euros is going to be paid (principal + interest) in 4.5 years. The interest rate is:
➢ 7.5% in the first year
➢ 5% in the second year
➢ 4% in the third year and following periods.
b) Find the total amount to be paid, using compound interest.

1.7) A principal of 15,600.00 Euros was invested in a certificate of deposit at 9% compound interest. Knowing that the future value is 20,500.00 Euros, find the term of this investment (in years, months, and days).

1.8) A principal of 14,000.00 Euros was invested in a financial application paying (compound interest):
➢ 8% for the last two years.
➢ 10% in the earlier years.
➢ Knowing that the future value is 21,734.70 Euros, find the term of this investment.

Part IIIA - Nominal, effective, and equivalent interest rates

1.10) Using compound interest, find the equivalent rates:
a) $i_Q = 2,1\%$ => $i_{4M} = ?$
b) $i_A = 6,4\%$ => $i_A^{(4)} = ?$

c) $i_A{}^{(2)} = 8\%$ $\Rightarrow I_S = ?$
d) $i_S = 4,7\%$ $\Rightarrow i_{4M} = ?$
e) $i_A{}^{(3)} = 9\%$ $\Rightarrow i_A = ?$
f) $i_Q{}^{(3)} = 1,8\%$ $\Rightarrow i_{4M} = ?$
g) $i_S{}^{(2)} = 6,2\%$ $\Rightarrow i_A{}^{(3)} = ?$

1.11) Using compound interest, find the _____ rate that is equivalent to a 4-month interest rate of 6%.

    a) Semiannual
    b) Quarterly
    c) 4-Month compounded monthly
    d) Annual compounded quarterly
    e) Monthly
    f) Annual instantaneous

1.12) Using compound interest, find the _____ rate that is equivalent to a semiannual interest rate of 8% compounded quarterly.

    a) 4-Month
    b) Annual
    c) Quarterly compounded monthly
    d) Annual compounded semiannually
    e) 4-month instantaneous

1.13) Sigma Enterprises made a 4-year investment under compound interest. The effective annual interest rate is 21% and interest is paid every six months.

    a) Find the effective quarterly interest rate of this investment.
    b) Find the nominal annual interest rate of this investment.
    c) Knowing that the interest of the second semester of the second year was 931.70 Euros, find the amount invested by Sigma Enterprises.

1.14) Mr. Tito wants to invest 50,000.00 in a certificate of deposit for 15 months. He is considering the following alternatives:

    - Bank X – quarterly payment of interest, with a nominal annual rate of 8%.
    - Bank Y – effective semiannual rate of 4%.
    Find:
    a) The future value for each alternative.
    b) The effective annual rate for each alternative.

1.16) Using compound interest, how long does it take for a principal of 1,500.00 Euros, with an effective annual rate of 7.5%, to have the same future value of a principal of 1,800.00 Euros, invested for the same time but with an effective annual rate of 5.0778%?

1.18) When Thomas was born, his grandfather bought him a 250 Euros savings bond that paid 5% per annum (compound interest). When John started college at age 18, he cashed in the savings bond. How much did John get?

1.20) On 1/09/2003 you borrow 5,000.00 Euros at $i_A^{(2)} = 6\%$ and on 1/3/2005 you pay 3,000 Euros. At that time what is your outstanding balance?

1.22) Natasha is considering an investment that she thinks will be worth 25,000.00 Euros in 10 years. What should she pay for this investment in she desires $i_A^{(4)} = 9\%$ on her money?

Part III B – Equation of Value

2.2) Mr Jones borrowed a certain amount of money at a semiannual rate of 5%. He agreed to pay back the loan in the following terms.

➢ 3,500.00 Euros after 3 months.
➢ 5,800.00 Euros after 9 months.
➢ 9,300.00 Euros after 15 months

Find the amount of money Mr. Jones borrowed considering:
a) Simple interest.
b) Compound interest.

2.3) Lambda Enterprises borrowed money at a 4-month interest rate of 3%. The loan is to be reimbursed in three payments of 3,000.00, 4,000.00, and 5,000.00 Euros, which are due, respectively, 1, 2, and 3 years after the loan was made.

However, 1,5 years after the initial loan contract, Lambda Enterprise decided to renegotiate the loan with the bank. The company had experienced some cash-flow problems in the former year and had not been able to make the first payment, but the current financial situation allows Lambda Enterprises to repay the total amount of the loan.

Find the amount of money Lambda Enterprises is going to pay the bank, considering:
a) Simple interest and moment 0 as the focal date.
b) Simple interest and 1,5 years as the focal date.
c) Compound interest.

2.4) Capa-Fi Enterprises borrowed 15,000.00 Euros at a 4-month interest rate of 4%. The reimbursement agreement is:

> 3,000.00 Euros after 4 months.
> 6,000.00 Euros after 8 months.
> The remaining after 20 months

Find the amount of the last payment considering:
   a) Simple interest.
   b) Compound interest.

2.5) Lusa Enterprises wants to invest 7,000.00 Euros for 3 years. The company is studying two investment alternatives, both considering compound interest:

   Bank Alpha is offering the following annual interest rates:
> First year: 5.5%
> Second year: 4.25%
> Third year: 3.75%

   Bank Beta offers an annual interest rate of 4.5% that remains the same for the three years.
   a) Which bank is offering the best deal?
   b) If Lusa Enterprises wants to obtain an interest of 1,103.38 Euros and an interest rate that remains the same for the three years, what should that interest rate be?
   c) If Lusa Enterprises chooses Bank Beta, how long will it take for the initial capital to double?
   d) If Lusa Enterprises chooses Bank Beta and wants to obtain a future value of 10,000.00 Euros in three years time, how much should the company invest today?

2.6) Psi Enterprises has a client that is supposed to pay the following amounts:
> 3,000.00 Euros on 1/1/2007.
> 10,500.00 Euros on 1/4/2007
> 4,500.00 Euros on 1/7/2008

Using compound interest and an annual interest rate of 15%:
   a) Find the equivalent capital on 1/10/2006
   b) If the client proposes to make one single payment of 16,500.00 Euros instead of the three payments, when should that payment occur?
   c) If the client pays 19,500.00 on 1/7/2007, what is the interest rate?

2.7) Omega Enterprises wants to invest 3,000.00 Euros for four years, and is offered the following interest rates:
> First year: 6%
> Second year: 5.25%
> Third year: 3.75%
> Forth year: 4.5%

Find:
   a) The future value using simple interest
   b) The future value using compound interest

c) The average interest rate that is equivalent to the four interest rates, using simple interest

d) The average interest rate that is equivalent to the four interest rates, using compound interest

2.8) Epsilon Enterprises has the following investment plan:
Find:
a) The future value (4 years from today) using simple interest.
b) The future value (4 years from today) using compound interest.
c) The average annual rate that is equivalent to the three rates, using simple interest.
d) Write the expression that would allow you to find the average annual rate that is equivalent to the three rates using compound interest.

2.9) At 8.0% per annum and simple interest, what one amount of money in one year is equivalent to 1,000 Euros now and €1,500 in 4 months?
a) Use today as the focal date.
b) Use 4 months as the focal date.
c) Use 1 year as the focal date.

2.10) Mrs Lemon owes 25,000.00 Euros in two years and 30,000.00 Euros in five years. What is the today cash equivalent of these two amounts if $i_A=4.1\%$ and compound interest is used?

2.11) Mr. Smith has two children, one is 5 years old and the other is 8 years old. Mr. Smith had 10,000.00 Euros which he divided into two separate accounts, both paying a 4-month interest rate of 6%. Each account was made for one of the children and each child is allowed to cash in the account on the day he/she becomes 18. Mr. Smith's objective is that the absolute value each child receives on his/her 18th birthday is exactly the same.
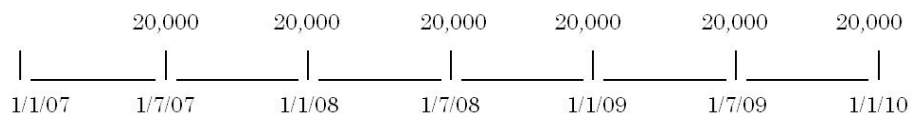
Assuming that the children were born on the same day of the same month and that the deposit was made on their birthday (when one became 5 and the other became 8), find:
a) The effective annual interest rate.
b) The amount invested into each account.
The amount the children receive on their 18th birthday

Part IV - Annuities

3.1 Consider the following annuity:

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            20,000      20,000      20,000      20,000      20,000      20,000

    | _____ | _____ | _____ | _____ | _____ | _____ |
    1/1/07     1/7/07     1/1/08     1/7/08     1/1/09     1/7/09     1/1/10
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Knowing that the semiannual interest rate is 1.5%:
  a) Find the present value on January 1, 2007.
  b) Find the future value on January 1, 2010.

3.29) Jan borrows 30,000.00 € at $i_A^{(2)}$=10% for her college education and wants to repay it within 10 years by making semiannual payments. Find the semiannual payment.

3.30) Preston is saving money to buy a motorcycle three years from now. He expects that the price will be 12,500.00€. He decides to make deposits at the end of each month. His savings account is paying $i_A^{(12)}$=4%. Find the monthly deposit.

3.32) Is it better to pay 24,000.00 € in cash or to pay 2,000.00 € per month for a year at $i_A^{(12)}$=12%?

3.33) Anna's car loan is costing her 500.00 € per month for five years at $i_A^{(12)}$=5.5%. Find the cash value of Anna's car.

3.39) Mrs. Lopes borrowed 14.000 Euros, which she will pay in 8 quarterly payments of 2.000 Euros. Knowing that the first payment is due on the day of the loan, write the equation that would allow you to calculate the quarterly interest rate of Mrs. Lopes loan.

3.34) Find the future value of 20 quarterly payments of 2,500.00 € each, at i_Q=3%.

3.35) Tony borrowed 50,000.00 € and will pay this amount back within the next five years. If his monthly payments are made at the beginning of each month, for i_M=0.25% find the monthly payment.

3.36) Find the monthly deposit to raise 2,500,000 € by age 65 if you make the first deposit at age 45 and you stop at age 60. Money is worth i_M=0.4%.

Part V - DEFERRED ANNUITIES

3.2) Rod-Alfa Enterprises is going to make 6 annual payments of 10,000€ each. Using compound interest and an effective semiannual rate of 2%, find the present value:
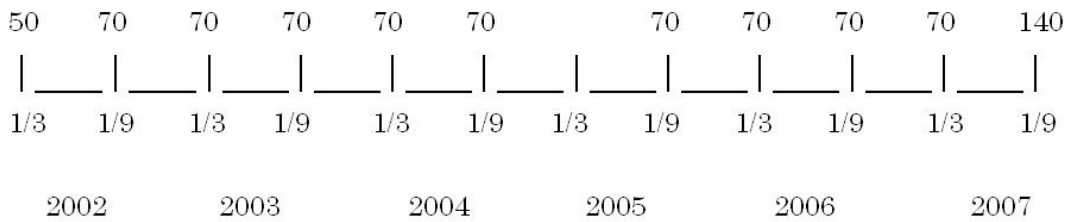 a) if the first payment is due a year from today
  b) if the first payment is due two years from today
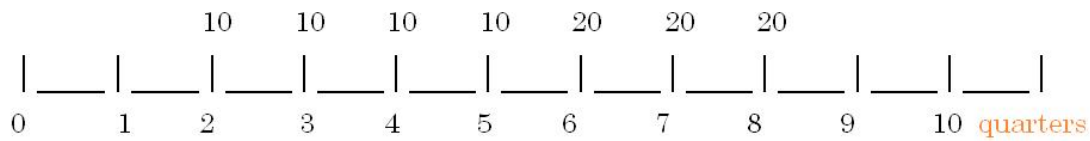
c) if the first payment is due today

3.3) Beta Enterprises will receive 8 annual payments of 800€. Each payment is due at the end of the year. Considering compound interest and an annual rate of 4%, find:
   a) The present value if the first payment is due one year from today
   b) The present value if the first payment is due three years from today
   c) The future value if the first payment is due one year from today
   d) The future value if the first payment is due three years from today

3.6) Find the value of the following group of capitals on March 1, 2002. $I_A^{(2)} = 12\%$.

50	70	70	70	70	70		70	70	70	70	140
1/3	1/9	1/3	1/9	1/3	1/9	1/3	1/9	1/3	1/9	1/3	1/9

2002   2003   2004   2005   2006   2007

3.7) Lusitana Enterprises owes the following amounts (thousand euros), with compound interest and a semiannual interest rate of 12.36% for the first year and 10.25% for the remaining periods.

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              10    10    10    10    20    20    20
 |____|____|____|____|____|____|____|____|____|____|
  0    1    2    3    4    5    6    7    8    9    10 quarters
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     a) Find the amount borrowed by Lusitana enterprises on the date of the loan contract (moment 0).

     b) Lusitana Enterprises only payed the first three amounts as scheduled. Ten quarters after the date of the loan contract, the enterprise payed the remaining amount of the debt, including interest. Find the amount Lusitana Enterprises paid.

3.8) For 5 years, K Enterprises has made monthly deposits of 500 Euros. These deposits earned a compound interest, the interest rate being 9% compounded monthly in the first two years, and 6% compounded monthly in the following years.
Find the future value of these deposits:
    a) Immediately after the last deposit.
    b) Immediately before the last deposit.

3.10) Omega Enterprises borrowed 90,000 Euros and is studying the following reimbursement alternatives:
    a) 15 payments, one every 4 months, the first payment is due 8 months after the contract, with a nominal annual interest rate of 9% compounded every 4 months.
    b) 6 annual payments, the first payment is due 15 months after the contract, with an effective semiannual rate of 5%.
    c) 20 quarterly payments, the first payment is due 9 months after the contract, with a nominal annual interest rate of 8% compounded quarterly.
    d) 60 monthly payments, the first is due on the date on the contract, with a nominal semiannual interest rate of 6% compounded monthly.

3.31) A 10,000.00 € loan for 36 months at $i_A^{(12)}=6\%$ is arranged for the customer to make monthly payments of R for the first year, 2R for the second year, and 3R for the third year. Find the payments for each of the three years.

3.20) Beta Enterprises borrowed money to buy a new equipment. They will pay back in 4 years, through quarterly payments of 450 Euros. The nominal annual interest rate is 8,0% compounded quarterly. Find how much the equipment costs, if:
    a) the first payment is due three months after the loan.

b) the first payment is due six months after the loan.

3.11) Mr. Santos borrowed 20,000 Euros, which he will reimburse in semiannual payments of 1,800 Euros each. The effective annual rate is 12,36%.Knowing that the first payment is due one year after the loan agreement, find the number of semiannual payments Mr. Santos will make. If you can't find an exact number, find an amount for the last payment so that the loan is fully reimbursed.

3.12) A loan will be reimbursed through 18 quarterly payments of 2,500 Euros each. The first payment is due six month after the loan contract. For the first two years, the semiannual interest rate compounded quarterly is 12%. For the following periods, the effective semiannual rate is 10.25%.
Find the value of the loan on the day the contract is signed.

3.27) Without calcuting the present value or the future value, find the two annuities that are equivalent.

 a. $8000 \; _{5|}\ddot{a}_{6|0.05} + 8000 \; a_{5|0.05}$
 b. $8000 \; S_{10|0.05} \, (1.05)^{-10}$
 c. $8000 \; _{6|}a_{5|0.05} + 8000 \; a_{5|0.05}$
 d. $4000 \; a_{10|0.05} + 4000 \; a_{10|0.05} + 8000 \, (1.05)^{-6} \, (1.05)$

3.28) John is a student of ISEG and decided to save a constant amount each month to support the tuition fees of a masters that he wants to attend. His savings will be deposited in an account under compound interest, under the following conditions:
• Annual interest rate with monthly capitalization of 6,0%;
• Deposits will be made on the first day of each month;
• The first deposit will be made in January 2013.
The master tuition fee of € 5,000.00 is payable on October 1, 2015, immediately after the last deposit.

a) Calculate the Answer monthly amount to be deposited in order to achieve the amount required to pay the fees.

b) Consider that from Jan. 1, 2015, the effective monthly interest rate changes to 0.75%. Calculate the value that John should deposit each month to achieve the amount required to pay the fees.

Part VI A PERPETUITIES

3.5) For a semiannual interest rate of 4%, find the cumulative present value of the following:
 1.   Annuity due, 6 semiannual payments of 90 Euros.

2. Ordinary annuity, 4 semiannual payments of 90 Euros, the first payment is deferred for 5 periods.
3. Perpetuity due, semiannual payments of 90 euros, the first payment is deferred for 9 periods.

3.21) Draw a time line representing the following quarterly payments. For a quarterly interest rate of 1,5 %, find the present value of:

$$250 \; {}_{4|}a_{4|i} + 250 \; {}_{24|}a_{\infty|i} + 250 \; {}_{12|}a_{16|i} - 250 \; {}_{24|}\ddot{a}_{4|i} + 250 \; {}_{8|}a_{4|i}$$

Part VI B PROGRESSIONS

3.13) Delta Enterprises recently acquired an equipment and is considering the following payment alternatives
- A – Six monthly payments, in growing arithmetic progression, the first is due one month from today. The value of the first payment is 500 Euros and the constant is 50. The nominal annual rate compounded monthly is 9.6%.
- B- Four quarterly payments, in decreasing arithmetic progression, the first payment is due three months from today. The value of the last payment is 800 Euros and the constant is 100 Euros. The nominal annual rate compounded quarterly is 10%.
- C – Five semiannual payments, in geometric progression, the first payment is due six months from today. The value of the first payment is 400 Euros and the factor is 1.25. The effective semiannual rate is 5%.
- Which is the best alternative (lowest present value)?

3.4) Petro-Psi Enterprises must pay the following capitals to bank QF:
- 2,000 Euros 6 months from today
- 3,000 Euros 1 year from today
- 4,000 Euros 1,5 years from today
- 5,000 Euros 2 years from toda

Considering compound interest and an effective annual rate of 4%:
a) If Petro-Psi wanted to pay all of its debts today, what would be the equivalent capital?
b) If Petro-Psi wanted to pay all of its debts two years from today, what would be the equivalent capital?
c) Solve b) using the formula for capitals in arithmetic progression.

3.15) Knowing that the present values of the following groups of capitals are equivalent for a nominal annual rate of 8% compounded quarterly, find the value of the last payment of the second group of capitals.
- Group A – Annuity due, 9 quarterly payments of 500 Euros. The first payment is deferred six months.

- Group B – 7 annual payments, the first is scheduled to happen two years from today. The payments grow in arithmetic progression when the first payment equals the constant.

3.16) Find the present value of 9 quarterly payments. The first payment is scheduled to occur 2 semesters from today. The value of the first payment is 60 Euros and the following payments grow in an arithmetic progression where the constant is 10. The nominal annual rate is 10% compounded quarterly.

3.12) Psi-Fi Enterprises borrowed 11.000 Euros, which will be reimbursed through 10 monthly payments, growing in an arithmetic progression with a constant of 50 Euros. The first payment is scheduled for 6 months from today. Knowing that the nominal annual rate compounded monthly is 12%, find the value of the last payment.

3.19) Mrs Benta borrowed 50.000 Euros which she will reimburse through 10 bimonthly payments growing in a geometric progression where the factor is 1.05. The first payment is scheduled to happen six months from today and the effective bimonthly rate is 1.5%.
    Find the value of the first payment

3.17) AutoSelf enterprises will have to make 7 payments, one every four months. The value of the first payment is 10.000 Euros and the following grow in geometric progression with a factor 1.1. The equivalent capital of this group of payments on February 1, 2004 is 58,110.36 Euros. Find the date when the last payment is scheduled to happen, knowing that the effective 8 months rate is 14.49%.

3.14) Greendelta Enterprises recently acquired an equipment and is considering the following payment alternatives:
- A – Eight monthly payments, in growing arithmetic progression, the first payment is due one month from today. The value of the first payment is 1000 Euros and the constant is 500.
- B – Immediate payment of 5,000 Euros, and an ordinary annuity, with 4 payments of 4,500 Euros, one payment every four months.
- C –Ordinary annuity, eight quarterly payments of 2,800 Euros, the first payment is deferred six months.
    Knowing that compound interest is used and the effective semiannual rate is 6% for the first year and 8% for the following periods, which is the best alternative (lowest present value)?

Part VII - Amortization of Debts

4.10) Stephanie borrowed money to buy a new motorcycle, which costs 10,000 €. She will make equal monthly payments for 4 years, with $i_A^{(12)} = 6\%$.

    a) Find the amount of Stephanie's monthly payment

    b) Write the first 5 lines of the amortization table

    c) Write the 24th line of the amortization table.

4.9) PC Alfa Enterprises bought a new warehouse, which was financed through a bank loan. PC Alfa will pay back the loan through 36 equal monthly payments, the first of which is due one month after the loan contract. The effective monthly interest rate is 0.60%.

Moment	PMT	Interest	Principal	Balance
0				
1		750,00		
2				
3				

    a) Fill in the first three lines of the amortization table

    b) Immediately after the 24th payment, the monthly interest rate changed to 0.50%. What is the amount owed by PC Alfa before this change?

    c) What is the new monthly payment?

4.6) Beta Enterprises borrowed money, with $i_A^{(12)} = 12\%$. The loan will be reimbursed through 9 monthly payments of 200.00 Euros. The first payment occurred on the date of the loan contract.

Due to a cash-flow problem, Beta Enterprises did not make the 9th payment as scheduled. The bank agreed that the missing payment (200.00) would be made, with interest, one year after the date of the loan contract

    a) What was the amount of the loan?

    b) What was the amount paid by Beta Enterprises at the end of the year?

4.3) Capa Enterprises borrowed 50,000.00 € with $i_A^{(4)} = 12\%$. The loan will be reimbursed through 5 equal quarterly payments, the first of which is scheduled to occur three months after the loan contract.

    a) Find the amount of the quarterly payments.

    b) Fill in the first two lines of the amortization table.

    c) Immediately after the second payment the interest rate changed to an effective quarterly rate of 4%. Find the new quarterly payment and fill in the last lines of the amortization table.

4.8) Mrs Perkins is buying a house which costs 50,000 €. She will pay back the mortgage through equal monthly payments, over a period of 20 years. The nominal annual rate compounded monthly is 6% and the first payment is scheduled to occur one month after the mortgage contract.
  a) Find the amount of the monthly payments.
  b) Fill in the first two lines of the amortization table.
  c) Immediately after the 12th payment the interest rate changed to $i_A^{(12)} = 4.5\%$. Find the new monthly payment.
  d) Immediately before the 24th payment, Mrs Perkins decided to pay 5,000 €. Find the new monthly payment (including the 24th).
  e) Immediately before the 120th payment, Mrs Perkins paied back the total amount she still owed the bank. Find the amount Mrs Perkins paid.

4.1A) Lambda Enterprises borrowed 25,000 Euros, with $i_A=10.25\%$. The loan will be reimbursed through 6 equal semiannual payments. The first payment is scheduled to occur 1 year after the loan contract.
  a) Find the amount of the semiannual payments.
  b) Fill in the amortization table.

4.2A) Psi-Delta Enterprises borrowed 70,000 Euros, which will be reimbursed through 4 equal semiannual payments. The effective semiannual interest rate is 5% is the first year, and 6% in the second year. The first payment is scheduled to occur 6 months after the loan contract.
  a) Find the amount of the semiannual payments.
  b) Fill in the amortization table.

5) On January 1st, 2013, Robert received a loan of 10.000 Euros from David, and agrees to repay it by 6 annual payments (end of each year). Assuming that the amortization is constant and with a semi-annual effective interest rate of 4,0%, complete the amortization schedule.

Moment	PMT	Interest	Principal	Balance

6) The company ANDERS borrowed from the Bank CIF a total of € 12,000. This loan will be paid off through 36 constant quarterly payments (capital + interest). The interest rate is the nominal annual of 8,0%. It is known that the first payment will occur one semester after the loan contract date.

a. Determine the amount of each payment of this loan.
b. Fill out the following lines of the amortization table (Euros).

Period (Quarter)	Debt at beginning of the period	Interest	Payment	Amortization	Accumulative Amortization	Debt at end of the period
1						
2						
36						
37						

Part VIII - Bonds

1) The coupon rate for a 1,000 € par value bond is 7.5%. Find the semiannual interest payment for this bond.

3) The coupon rate for a 1,000 € par value bond is 7.5%. Find the semiannual interest payment for this bond.

4) A government bond with 12 years to maturity carries a coupon rate of 6%. Similar government bonds with the same remaining term sell on the open market with a 7.1% interest rate. Would you expect the bond to sell at a premium or at a discount?

7) An investor purchased a bond for €4,875.35 - The par value is €5,000 and the coupon rate is 6.75% with annual payments. Find the current yield.

9) 1,000€ par value bond is sold at a 45.36€ discount. The coupon rate is 5.75%. What is the current yield?

11) Suppose you are considering investing some money in government bonds. You look up a rate quote on your broker's web site, and get the following quote:

Par value	Coupon (%)	Maturity	Current yield	Last price (%)
10,000	6.25	31/10/2012		103.516

   a) What is the selling price for one of these bonds?
   b) There was a mistake and the current yield was left blank. What should have been on that spot of the table?

16) Edgar has quite a bit of money invested in the bonds of the Port Gibson Widget Company. He hears on the morning business news that a major rating agency has upgraded the company's credit rating. On the basis of these news, would you expect that each of the following would increase, decrease, or remain the same?
   a) The market value of his bonds.
   b) The current yield of these bonds
   c) The par value of his bonds
   d) The semiannual interest payment on his bonds

17) Sybil owns several U.S. Treasury bonds. On the radio she years a well-known financial commentator predict that interest rates are going to rise significantly over the

next few months. Would this be good news or bad news for the market value of her investments?

23) Eighteen years ago, Porshia's grandmother bought her a 1,000 € face value savings bond for 500 € . Today, she can cash the bond in for 1,514.67 €. What is the effective interest rate earned by this bond (CARG)?

24) A bond has a face value of 4.000, an annual coupon rate of 4% and matures 4 years from today   Knowing that the yield to maturity is 2%, what should the purchase price be?
You buy 4 of the bonds mentioned. Fill in the amortization table of your investment.

25)   A bond has a face value of 4,000, an annual coupon rate of 4% and matures 4 years from today   Knowing that the yield to maturity is 6%, what should the purchase price be? You buy 4 of the bonds mentioned in **a).** Fill in the amortization table of your investment.

26) On May 3, 2004 Valera bids 96,2 on 182-day $500.000 T-bill.

a)   Find the implicit discount and as an interest rate.

b)   On august 5, 2004 Vaelra sells the T-Bill to a women desiring a 6% simple interest rate eturn on her investment. Find the amount the women paid for the T-bill on a simple interest basis.

27) The company BONDS decided to issue a bond loan, the conditions were as follows:

• Date of issue: 01/03/N.
• Nominal value: 12,00 Euros (par)
• Issued at par
• Number of Bonds: 800 000.
• Loan term: 3 years.
• Semi-annual coupon rate of 6,0%.
• Payment of semi-annual interest.
• First interest payment: 6 months after issuance.
• Mode of redemption: Repayment of bonds in 2 equal batchs (number of bonds).
 • Date of first redemption: two years after issuance.
• Date of second redemption: three years after issuance.
• Redemption premiums € 0.40 (1st redemption) and € 0.80 (2nd redemption)

a) Fill out the following lines for the amortization table for this bond loan.

Period (Quarter)	Debt at beginning of the period	Interest	N.º of redemption bonds	Amortization	Redemption Premium	Total payment (interest+amortization)
1						
2						
3						
4						
5						
6						

b) Consider that an investor purchased 400 bonds of this loan in the next day after the first redemption. The second batch of bonds was reimbursed, and the investor obtained an effective semiannual rate of return of 6.50%. Compute the value that the investor paid for each of those bonds.

28) A bond with face value €10.000 matures 4 years after issue. Interest payments are made semiannually with effective semiannual rate of 5%. Build the amortization table for the bond over its term for a yield to maturity annual nominal rate compounded semiannually of:
a) 8%
b) 10%
c) 12%.

Part IX – Leasing

2) Hugo is looking at a new truck costing 18,345 €. Calculate his monthly payment:
   a) Assuming he buys the truck, financing the full price with a 5-year loan at $i_A^{(12)}$ = 9%.
   b) Assuming he leases the car for 3 years. Assume a 12,910 € residual value and $i_A^{(12)}$ = 9%.

3) Vanessa is looking at leasing a new car for two years. The total cost is 24,075 €, the residual value would be 18,950 € and $i_A^{(12)}$ = 7.5%. Calculate her monthly lease payment
   a) Assuming she makes no down payment
   b) Assuming she makes a 3,500 € down payment.

4) A car dealer runs an ad for 3-year leases on two different car models. The cost for a new Cascadia sedan is 31,953€ and the 3-year residual value is 25,750€. The cost for a new Kiriana sedan is 27,935€ and the 3-year residual value is 15,910€. Calculate the lease payment for each of these vehicles assuming $i_A^{(12)}$ = 8%. Is there anything surprising about your results?

10) A college needs a new photocopier, which costs 4,575 €. It is considering a lease for the copier. An office equipment supply company offers a 3-year lease with a limit of 25,000 copies per year. Each copy above this limit would be charged 3.5 cents. If the college takes this lease and makes 93,585 copies over the 3 years of the lease, how much extra would it have to pay?

13) Brandon wants a new car costing 19,992 € . The 2-year residual value is 10,052 and $i_A^{(12)}$ = 7.5%. Brandon will trade in his current car, worth 8,500 €. Calculate his monthly lease payment.

15) In exercise 4 we calculated the lease payment for two different cars. Suppose the dealer complains to the manufacturer that the lower-priced car has a higher payment and asks for a reduction in the price of the new Kiriana so that its lease payment would be the same as for the Cascadia. What would the cost of the Kiriana need to be to make this happen?

17) Suppose you have a 2-year lease on a car that allows for 14,000 miles per year. The overmileage penalty is 37.5 cents per mile. In the first year you put 11,864 miles on the car, and in the second year you put 15,661 miles on it. How much of a mileage penalty, if any, will you owe?

18) The Quick-Link company is looking at leasing new equipment. The terms of the leasing contract are as follows:
• No initial entry;
• Term of contract: 5 years;

• Monthly payments. The first payment will be at the end of first month. The payments of the first 2 years are constant and have half the value of the constant payments in the last three years;
• Nominal annual rate of monthly capitalization: 10.2%;
• Residual value of € 300,00, to be paid one month after the last payment.

a) Assume that the value of the equipment is € 10,989.30. Calculate the value of the lease payments in the first 2 years and in the last 3 years.

b) Assuming the lease payments are constant and equal to € 250.00 throughout the term of the contract, with the same residual value of € 300,00, what would be the value of the equipment?

19) You are planning to buy a new car. You find you can buy a good car for €22.000, but you do not have that much money available. You have two financing choices:
1) You get a 5-year loan at 6,0% interest compounded monthly;
2) You can lease the car, for that you pay €1.500 right away when you sign the lease, then €265 each month for 5 years. And you would also have the buy the car for €12.000 in 5 years, when the lease is over.
What is the better option for you?

Assorted Exercises

1) McGill plans to create a scholarship fund that will pay 50 students a monthly spend of €200 at the beginning of months September through April, plus an amount of €300 on the following May 1st.

(a) If interest is assumed to be at a nominal annual rate of 6% per annum, compounded monthly, determine the amount that is needed in this fund on September 1st just before the fund begins making payments. Consider one year of payments.

(b) Determine the amount that will be in the fund just after the December scholarship payments in the first year.

2) The company VL, SA intends to get a loan of 350000 Euros payable in 10 years. The company went to two banks, the Bank AB and the Bank KU.

Bank AB Proposal:
• Annual interest rate of 5%.
• Constant annual payments, immediate.

Bank KU Proposal:

• Quarterly interest rates of 2% during the first five years of the contract and the remaining 3%.
• Constant annual payments, immediate.

a) Compute the annual payment associated with each option. Which option is more advantageous for the company?

b) Assume that the company opted by the Bank AB proposal and at the end of the fifth year of the contract made a capital amortization of 20% of principal outstanding at that time. Calculate the amount of this capital amortization.

3) The Football Club Fruit decided to issue a bond loan to purchase new players. The issue conditions were as follows:
• Date of issue: 31/05/2013.
• Face value: 10.00 Euros.
• Number of Bonds: 120 000.
• Issue price: 7.5 Euros
• Loan term: 6 years.
• Annual coupon rate of 4.0%.
• Payment of annual interest immediate.
• First interest payment: 1 year after issuance.
• Refunds of a constant number of bonds with biannual (every two years), starting at the end of the 2nd year.

a) What value of this operation for the club?

b) What is the value of accumulated amortization at the end of the fourth year?

c) Consider that a club member purchased 300 bonds on the issue date, having been reimbursed at the end of the loan. Write the equation for calculating the yield corresponding.

4) The D. Pilar built a pool using a lease contract with the following conditions:
• Duration of contract: 3 years.
• quarterly effective interest rate of 2.3%.
• System of payments: quarterly payments. The first payment is paid after 8 months contract.
• Initial Input: 12% of the total pool cost.
• Salvage: settled with the last payment and the amount of 10% of the contract value, assuming the value of 500 Euros at the end of it.

a) What is the value of the pool?
b) Determine the amount of each payment.

c) What is the amount of the debt at the end of the second year of the contract?

5) John is a student of ISEG and decided to save a constant amount each month to support the tuition fees of a masters that he wants to attend. His savings will be deposited in an account under compound interest, under the following conditions:
• Annual interest rate with monthly capitalization of 6.0%;
• Deposits were made on the first day of each month;
• The first deposit was made in January 2013.
The master tuition fee of € 5000.00 is payable on October 1, 2015, immediately after the last deposit.
Answer the following questions:
a) Calculate the monthly amount to be deposited in order to achieve the amount required to pay the fees.

b) Consider that from Jan. 1, 2015, the effective monthly interest rate changes to 0.75%. Calculate the value that John should deposit each month to achieve the amount required to pay the fees.